

## 1. Conditional Distributions of Discrete Random Variables

Suppose  $X$  &  $Y$  are two discrete r.v.'s with joint PMF  $p(x, y)$  and marginal PMF's  $p_X(x)$  and  $p_Y(y)$  respectively.

The conditional PMF for  $Y$  given  $X = x$  is

$$P_{Y|X}(y|x) = \frac{P(X = x, Y = y)}{P(X = x)} = \frac{p(x, y)}{p_X(x)}$$

The conditional PMF for  $X$  given  $Y = y$  is

$$P_{X|Y}(x|y) = \frac{P(X = x, Y = y)}{P(Y = y)} = \frac{p(x, y)}{p_Y(y)}$$

## 2. Conditional Distributions of Continuous Random Variables

Suppose  $X$  &  $Y$  are two continuous r.v.'s with joint PDF  $f(x, y)$  and marginal PDF's  $f_X(x)$  and  $f_Y(y)$  respectively.

The conditional PDF for  $Y$  given  $X = x$  is

$$f_{Y|X}(y|x) = \frac{f(x, y)}{f_X(x)}$$

The conditional PDF for  $X$  given  $Y = y$  is

$$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)}$$

## 3. Conditional = Marginal when independent

The conditional distribution of  $Y$  given  $X = x$  if  $X$  and  $Y$  are independent:

$$f_{Y|X}(y|X = x) = \frac{f_{X,Y}(x, y)}{f_X(x)} = \frac{f_X(x)f_Y(y)}{f_X(x)} = f_Y(y)$$

i.e. the conditional PDF  $Y|X$  is the marginal PDF of  $Y$ .

In fact, the following three are equivalent definitions of the independence of  $X$  and  $Y$ :

1.  $f(x, y) = f_X(x)f_Y(y)$ , i.e. joint = product of marginal
2.  $f_{Y|X}(y|X = x) = f_Y(y)$ , i.e. conditional  $Y|X$  = marginal of  $Y$
3.  $f_{X|Y}(x|Y = y) = f_X(x)$ , i.e. conditional  $X|Y$  = marginal of  $X$

These facts apply to joint/conditional/marginal PMFs for discrete,  $X, Y$ , too.

## 4. EV of a Random Variable

Relatively trivial stuff,  $\sum_x xp(x) = \int xdx$  moreorless.