

1. Conditional Distributions of Discrete Random Variables

Suppose X & Y are two discrete r.v.'s with joint PMF $p(x, y)$ and marginal PMF's $p_X(x)$ and $p_Y(y)$ respectively.

The conditional PMF for Y given $X = x$ is

$$P_{Y|X}(y|x) = \frac{P(X = x, Y = y)}{P(X = x)} = \frac{p(x, y)}{p_X(x)}$$

The conditional PMF for X given $Y = y$ is

$$P_{X|Y}(x|y) = \frac{P(X = x, Y = y)}{P(Y = y)} = \frac{p(x, y)}{p_Y(y)}$$

2. Conditional Distributions of Continuous Random Variables

Suppose X & Y are two continuous r.v.'s with joint PDF $f(x, y)$ and marginal PDF's $f_X(x)$ and $f_Y(y)$ respectively.

The conditional PDF for Y given $X = x$ is

$$f_{Y|X}(y|x) = \frac{f(x, y)}{f_X(x)}$$

The conditional PDF for X given $Y = y$ is

$$f_{X|Y}(x|y) = \frac{f(x, y)}{f_Y(y)}$$

3. Conditional = Marginal when independent

The conditional distribution of Y given $X = x$ if X and Y are independent:

$$f_{Y|X}(y|X = x) = \frac{f_{X,Y}(x, y)}{f_X(x)} = \frac{f_X(x)f_Y(y)}{f_X(x)} = f_Y(y)$$

i.e. the conditional PDF $Y|X$ is the marginal PDF of Y .

In fact, the following three are equivalent definitions of the independence of X and Y :

1. $f(x, y) = f_X(x)f_Y(y)$, i.e. joint = product of marginal
2. $f_{Y|X}(y|X = x) = f_Y(y)$, i.e. conditional $Y|X$ = marginal of Y
3. $f_{X|Y}(x|Y = y) = f_X(x)$, i.e. conditional $X|Y$ = marginal of X

These facts apply to joint/conditional/marginal PMFs for discrete, X, Y , too.

4. EV of a Random Variable

Relatively trivial stuff, $\sum_x xp(x) = \int x dx$ moreorless.